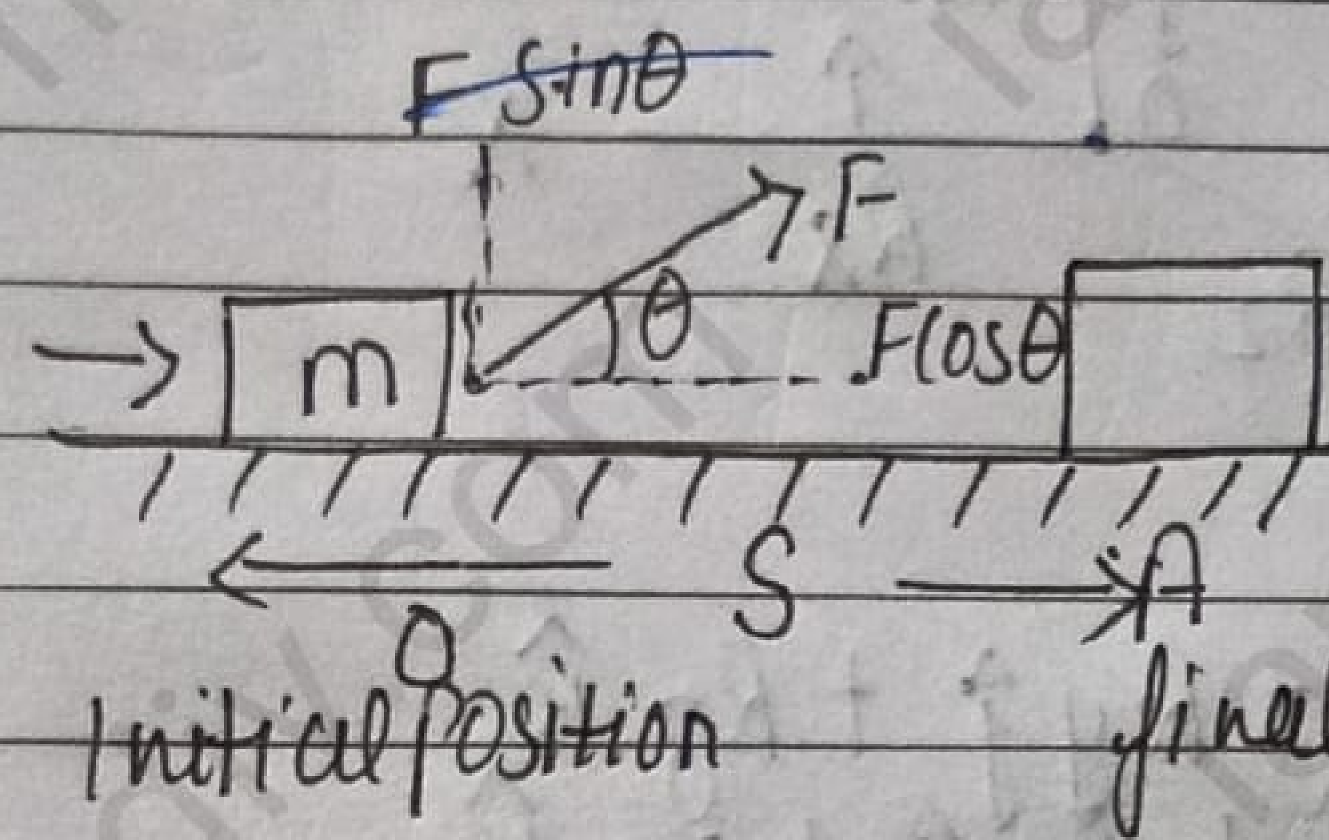
11/Aug/2023

Chapter - Work, power and Energy

Force → Push or Pull



Along to X-axis

$W = (F_{\text{Along to x}})$ Displacement

$W = (F\cos\theta)\, S$

$(\because \vec{A}\cdot\vec{B} = AB\cos\theta)$

$W = \vec{F}\cdot\vec{S}$

↳ Scalar Quantity

↳ Two vector, Dot product

$\boxed{W = \vec{F}\cdot\vec{S}}$ or $\boxed{W = F\cos\theta}$

If $\theta = 0°$, $W = FS$, $W = +ve$

If $\theta = 90°$, $W = 0$

If $\theta > 90$, $W = -ve$

Work done → +ve, -ve, 0

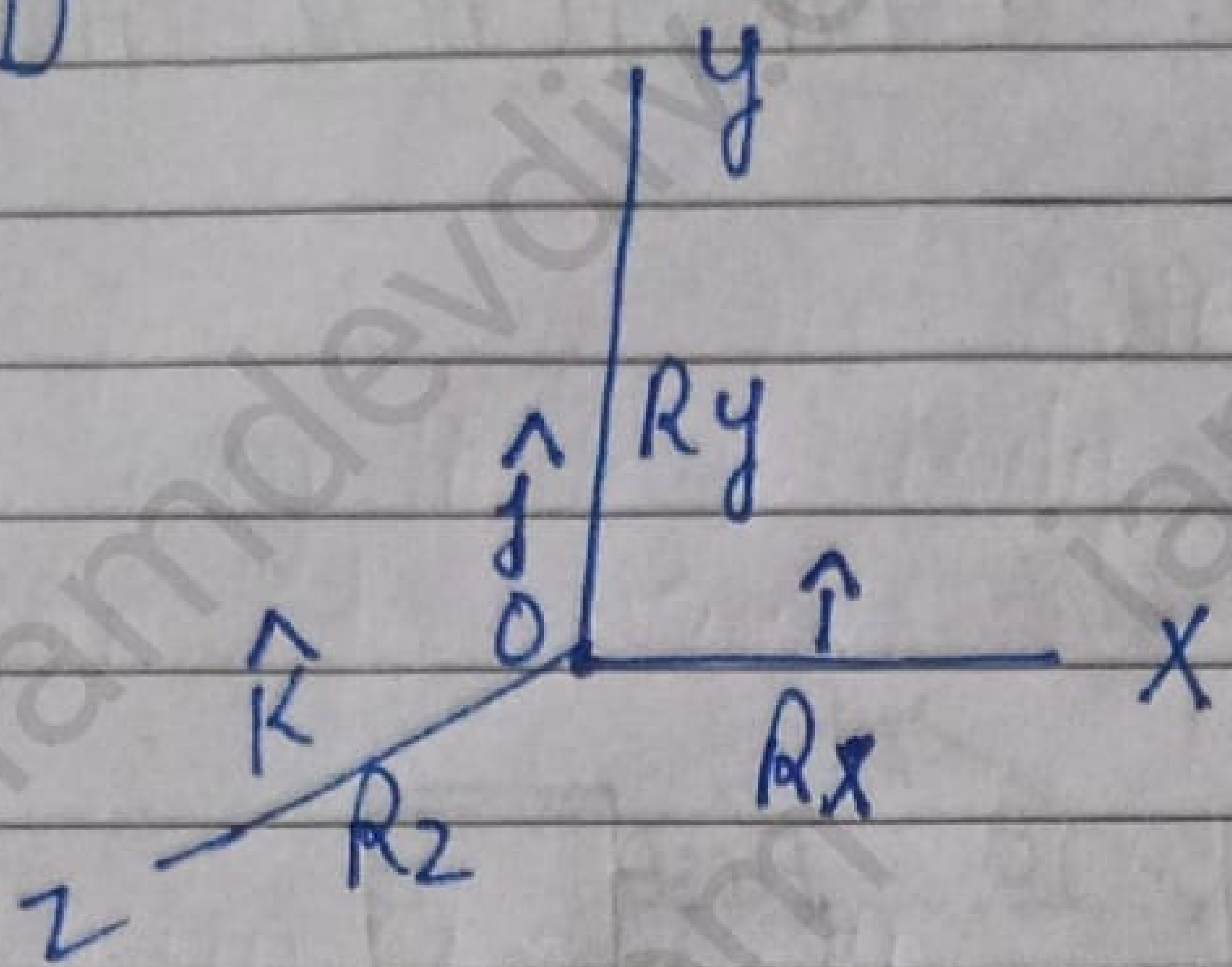
$W = FS$ = Newton. Meter

W = Joule → S.I./M.K.S.

W = Earg → C.G.S.

$1\,J = 10^7$ earg

$\vec{R} \to 3D$



$$\vec{R} = R_x\hat{i} + R_y\hat{j} + R_z\hat{k}$$
$$\vec{F} = \cancel{R}\, F_x\hat{i} + F_y\hat{j} + F_z\hat{k}$$

$$W = \vec{F}.\vec{S}$$
$$dW = F dx$$
$$\int dW = \int_{x_1}^{x_2} F dx$$

<u>Ques1-</u> A position dependent force $F = 7 - 2x + 3x^2$ act on a small body of mass 2 kg and displaced from 0 to $x = 5m$ Calculate work done in joule.

$$x_1 \to x_2$$
$$0 \to 5$$

$$\int dW = \int_{x_1}^{x_2} F dx$$

$$\int_0^W dW = \int_0^5 (3x^2 - 2x + 7) dx$$

$$W = 3\int_0^5 x^2 dx - 2\int_0^5 x\, dx + 7\int_0^5 1\, dx$$

$$= \cancel{3}\left[\frac{x^3}{\cancel{3}}\right]_0^5 - \cancel{2}\left[\frac{x^2}{\cancel{2}}\right]_0^5 + 7\left[x\right]_0^5$$

$$= (5^3 - 0) - (5^2 - 0) + 7(5-0)$$

$$= 125 - 25 + 35$$

$$= 160 - 25$$

$$= \boxed{135 \text{ Joule}}$$

Q2- $F = 6 - 4x + 6x^2$ $\qquad 6x^2 - 4x + 6$

$x = 0$ to $x = 2m$

$$\int_0^w dw = \int_0^x (6x^2 - 4x + 6)\,dx$$

$$W = 6\int_0^2 x^2\,dx - 4\int_0^2 x\,dx + 6\int_0^2 dx$$

$$W = \cancel{6}^{2}\left[\frac{x^3}{\cancel{3}}\right]_0^2 - \cancel{4}^{2}\left[\frac{x^2}{\cancel{2}}\right]_0^2 + 6\left[x\right]_0^2$$

$$W = 2(2^3 - 0) - 2(2^2 - 0) + 6(2-0)$$

$$W = 2\times 8 - 2\times 4 + 12$$

$$W = 16 - 8 + 12$$

$$W = 8 + 12$$

$$\boxed{W = 20 \text{ Joule}}$$

Ques Calculate the work done by the force $3\hat{i} - 2\hat{j} + 4\hat{k}$ in carrying a particle from $(-2m, 1m, 3m)$ to point $(3m, 6m, -2m)$

$(\underset{x_1}{-2m}, \underset{y_1}{1m}, \underset{z_1}{3m})$ $\qquad (\underset{x_2}{3m}, \underset{y_2}{6m}, \underset{z_2}{-2m})$

$$\vec{r} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$$

$$\boxed{\vec{r} = 5\hat{i} + 5\hat{j} - 5\hat{k}}$$

Energy

Energy is defined as the internal capacity to do work when we say that a body has energy, it means that it can do work.

Different form of energy
- Mechanical energy
- Sound energy
- Molecular, Atomic and nuclear energy etc.
- optical energy
- Electrical energy

Note:- These forms of energy can change from one form to another.

Mass Energy relation

According to Einstein mass energy relation.

$$\boxed{E = mc^2}$$

$1eV = 1.6 \times 10^{-19}$ J

$1cal = 4.2$ J

$1 KWh = 36 \times 10^5$ J

Kinetic Energy

K.E. is the internal capacity of doing work done of an object by virtue its motion

OR

K.E. of a body can be calculated by the amount of work done in stopping the moving body or by the amount of work done imparting the present velocity of the body from the state of rest.

Page No.
Date: / /20

If a particle of mass 'm' is moving with velocity $\vec{v}$ much less than the $\vec{v}$ of light then the

$$\boxed{K.E. = \frac{1}{2}mv^2}$$

$$v^2 = u^2 + 2as$$
$$v^2 = 0 + 2as$$
$$s = \frac{v^2}{2a}$$
$$W = FS$$
$$W = m\cancel{a} \times \frac{v^2}{2\cancel{a}}$$
$$W = \frac{1}{2}mv^2$$
$$\cancel{W}\ \boxed{K.E. = \frac{1}{2}mv^2}$$

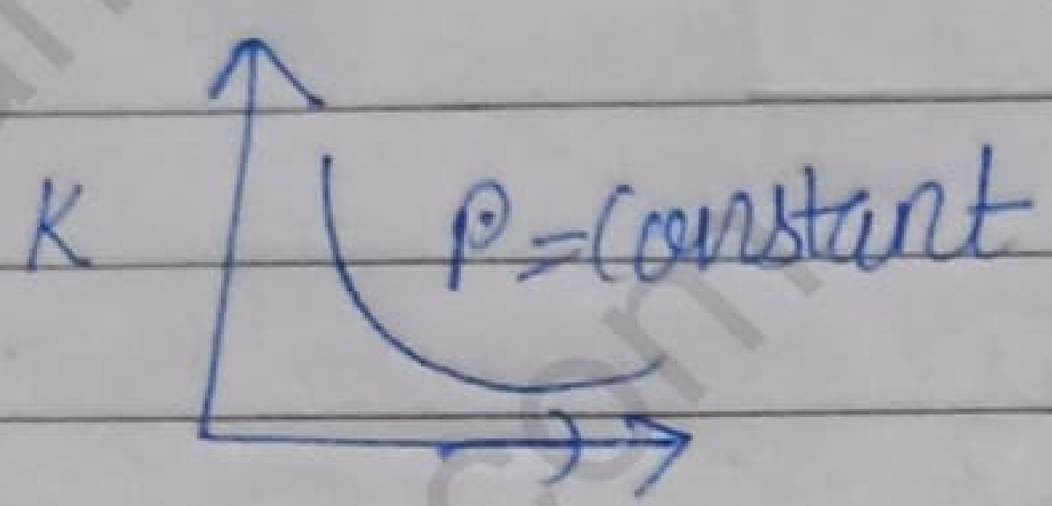
<u>Note:-</u> We know that Linear Momentum $= p = mv$ — (1)

$$K.E. = \frac{1}{2}mv^2 \quad \text{— (2)}$$

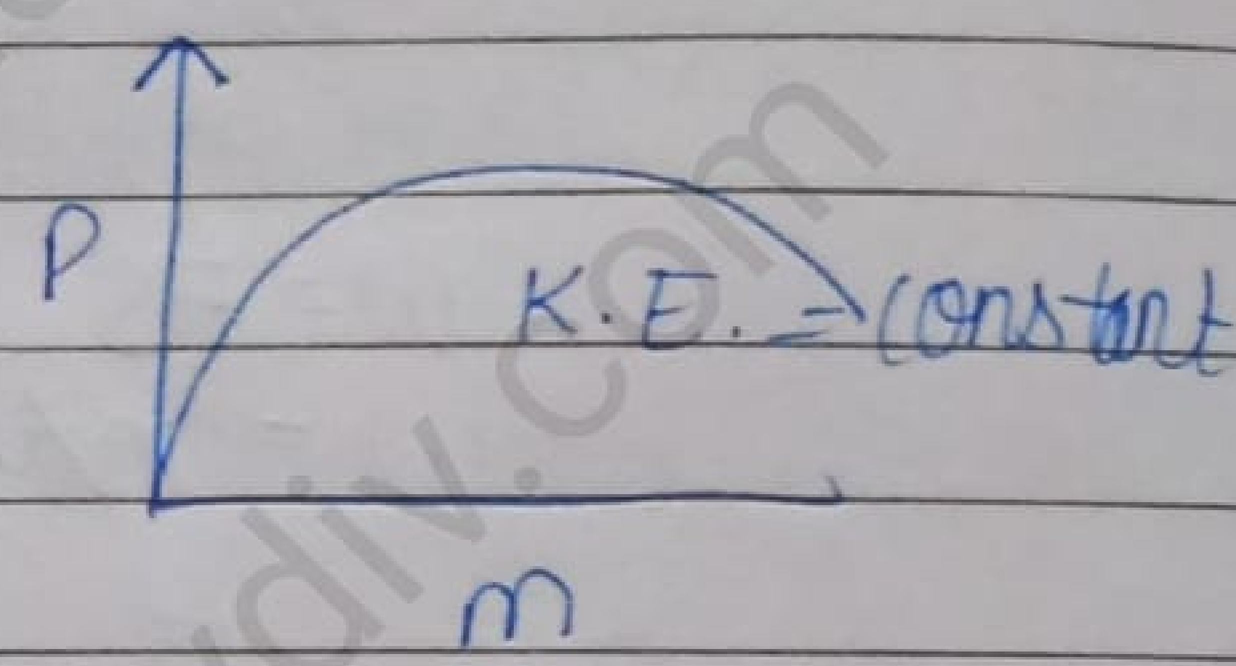
$$p = mv$$
$$p^2 = m^2v^2$$
$$\frac{p^2}{m} = mv^2$$
$$\boxed{\frac{p^2}{2m} = \frac{1}{2}mv^2}$$
$$\frac{p^2}{2m} = E$$
$$p^2 = 2mE$$
$$\boxed{p = \sqrt{2mE}}$$

If $p =$ Constant

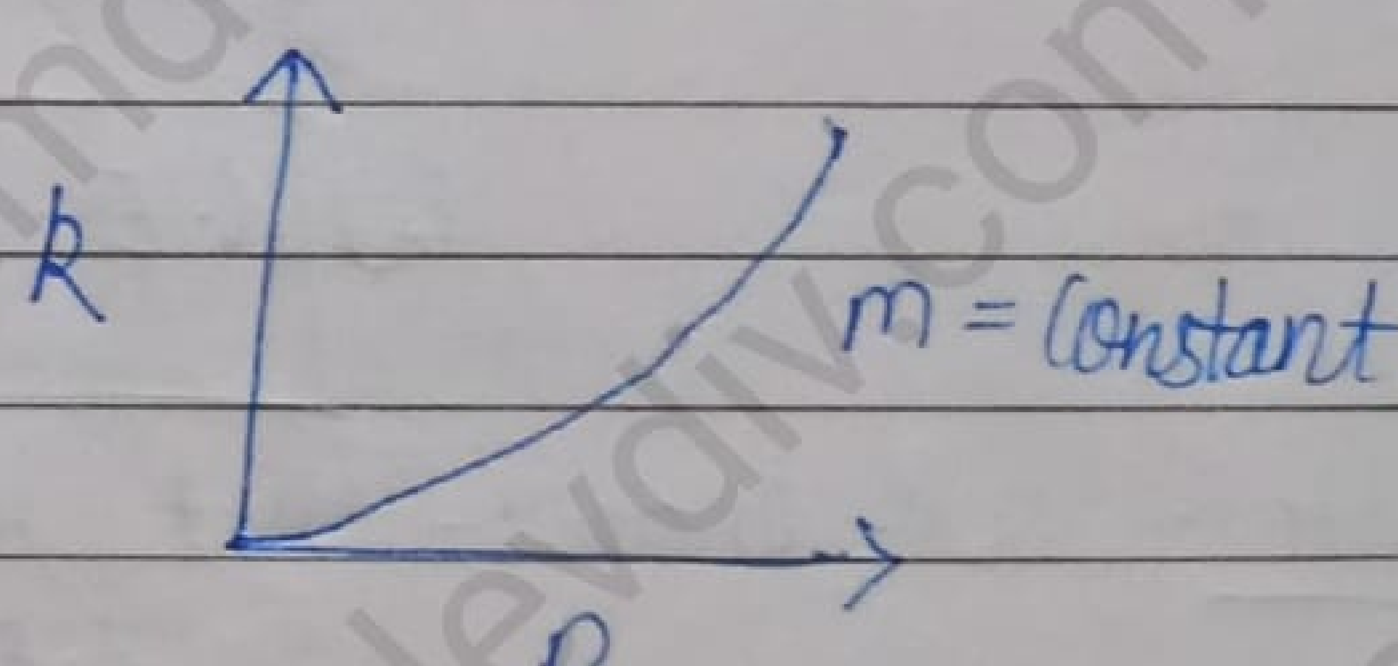
$$m \propto \frac{1}{\sqrt{K}} \qquad K = K.E.$$



If K.E = Constant $\quad P \propto \sqrt{m}$



If m = Constant $\quad P \propto \sqrt{K}$



Explain Work Energy Theorem

Work done by all the forces like conservative / non-conservative force or external / internal forces acting on a particle for an object is equal to the change in its K.E. so work done by all the forces = Change in K.E.

$$W = \Delta K.E.$$

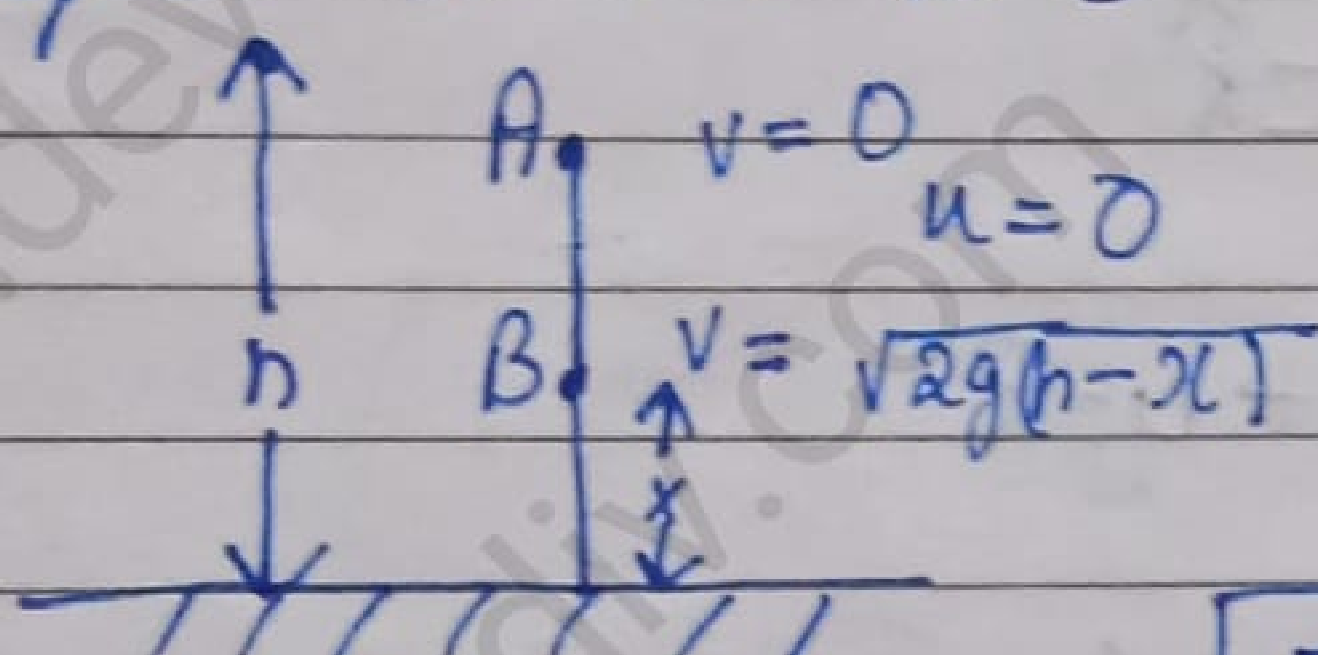
$$W = \frac{1}{2}mv_2^2 - \frac{1}{2}mv^2$$

Let us consider a mass particle at point O with velocity u after applying a force F, its velocity becomes v.

Mechanical Energy Conservation:-

\# $P.E. = mgh$
\# $K.E. = \frac{1}{2}mv^2$

\# $v^2 = u^2 + 2as$
\# $v^2 = u^2 + 2gh$
\# $v^2 = 2gh$
\# $M.E = K.E. + P.E.$
☆ when object drop A to C



$AC = h$
$BC = x$
$AB = h - x$

$\rightarrow v = \sqrt{2gh}$

$$\boxed{M.E._C = M.E._B = M.E_A}$$

At point C $K.E. = \frac{1}{2}mv^2 = \frac{1}{2}m(\sqrt{2gh})^2 = \frac{1}{2}m \times 2gh = mgh$

$M.E_C = K.E + P.E = mgh + 0 = mgh$ —(1)

At point B $K.E. = \frac{1}{2}mv^2 = \frac{1}{2}m(\sqrt{2g(h-x)})^2$

$= \frac{1}{2}m\,2g(h-x) = \frac{1}{2}mg(h-x)$

$P.E. = mgx$

$M.E_B = KE + P.E = mgx + mg(h-x) = mgh$ —(2)

At point A

$P.E = mgh$; $K.E. = 0$.

$M.E_A = P.E + K.E = mgh + 0 = mgh$ —(3)

eq (1) = eq (2) = eq 3

$$\boxed{M.E_C = M.E_B = M.E_A}$$

Note:- The relation between conservative ~~force~~ filled and potential energy

$$F = -\overset{\overset{\text{gradient}}{\uparrow}}{\text{grad}}.\ U$$

$$\underset{\text{Force}}{\underset{\downarrow}{F}} = -\frac{du}{dr} \begin{array}{l} \rightarrow \text{Potential Energy} \\ \rightarrow \text{distance} \end{array}$$

$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

$$u = u_x\hat{i} + u_y\hat{j} + u_z\hat{k}$$

$$\frac{du}{dr} = \frac{du}{dx}\hat{i} + \frac{du}{dy}\hat{j} + \frac{du}{dz}\hat{k}$$

Note:- In the Stable Equilibrium condition

Ques- Force between the atom of diatomic molecules has its origin in the interaction between the electron and the nuclei present in each atom This force is conservative and the associated potential energy $U(r)$ is good a approximation represente by following function

$$U(r) = U_0\left[\left(\frac{a}{r}\right)^{12} - \left(\frac{a}{r}\right)^{6}\right]$$

Where r is the distance b/w two atoms and U_0 and are positive constant develop expression

Page No.
Date: / /20

for assosiate force & ~~fitted~~ find the equilibrium separation b/w the atoms.

$\frac{d}{dr} r^n = n r^{n-1}$

$$U = U_0 \left[\left(\frac{a}{r}\right)^{12} - \left(\frac{a}{r}\right)^{6}\right]$$

$$\because F = -\frac{dU}{dr} = -U_0 \frac{d}{dr}\left[a^{12} r^{-12} - a^6 r^{-6}\right]$$

$$-\frac{du}{dr} = -U_0\left[a^{12}\frac{d}{dr} r^{-12} - a^6 \frac{d}{dr} r^{-6}\right]$$

$$= -U_0\left[a^{12}(-12 r^{-13}) - a^6(-6 r^{-7})\right]$$

$$= -\frac{U_0}{a}\left[a^{13}(-12 r^{-13}) - a^7(-6 r^{-7})\right]$$

$$= -\frac{U_0}{a}\left[-12\left[\frac{a}{r}\right]^{13} + 6\left(\frac{a}{r}\right)^7\right]$$

$$F = +\frac{6U_0}{a}\left[2\left(\frac{a}{r}\right)^{13} - \left(\frac{a}{r}\right)^7\right] \quad - (1)$$

$F = 0$ for stable equilibrium condition

$$\frac{6U_0}{a}\left[2\left(\frac{a}{r}\right)^{13} - \left(\frac{a}{r}\right)^7\right] = 0$$

$$2\left(\frac{a}{r}\right)^{13} - \left(\frac{a}{r}\right)^7 = 0$$

$$2\left(\frac{a}{r}\right)^{13} = \left(\frac{a}{r}\right)^7$$

$2 = \frac{a^7}{r^7} \times \frac{r^{13}}{a^{13}}$

$2 = \frac{r^6}{a^6}$

$r^6 = 2a^6$

$\boxed{r = 2^{1/6} a}$

Evergreen
Page No.
Date: / /20

21/Aug./2023

Force

$F = -\frac{du}{dr}$ - Potential energy

Numerical

$U = ax^3 - bx^2$ — (1)

$\frac{du}{dr} = \frac{du}{dx}\hat{i} + \frac{du}{dy}\hat{j} + \frac{du}{dz}\hat{k}$

$F = ?$

$F = -\frac{du}{dx}$

~~U = a~~ Equilibrium Condition U = ?

$F = -\frac{du}{dy}$

$F = 0$

$\boxed{\frac{d}{dx}x^n = nx^{n-1}}$

$F = -\frac{du}{dz}$

$U = ax^3 - bx^2$ — (1)

$\cancel{F = ?}$

$F = -\frac{d}{dx}u$ — (2)

Taking differentiate with respect to x

$\frac{du}{dx} = \frac{d}{dx}(ax^3 - bx^2) = a\frac{d}{dx}x^3 - b\frac{d}{dx}x^2$

$\frac{du}{dx} = 3ax^2 - 2bx$ — (3)

from (2) & (3)

$F = -3ax^2 + 2bx$ — (4)

$$F = -3ax^2 + 2bx \quad - (4)$$

For equilibrium stable condition

$$F = 0$$
$$-3ax^2 + 2bx = 0$$
$$3ax^2 = 2bx$$
$$\boxed{x = \frac{2b}{3a}} \quad - (5)$$

Putting value of x in eq (1)

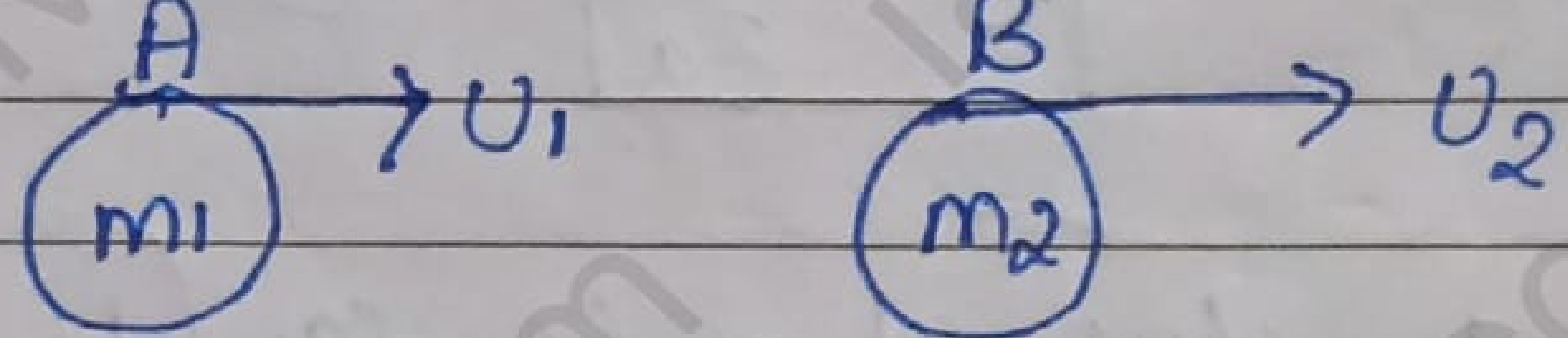
$$U = ax^3 - bx^2$$
$$U = a\left(\frac{2b}{3a}\right)^3 - b\left(\frac{2b}{3a}\right)^2$$
$$U = a\left(\frac{8b^3}{27a^3}\right) - b\left(\frac{4b^2}{9a^2}\right)$$
$$U = \frac{8ab^3}{27a^3} - \frac{4b^3}{9a^2}$$

Elastic Collision in 1-D

- → K.E. Conservation
- → Linear momentum Conservation

Case I

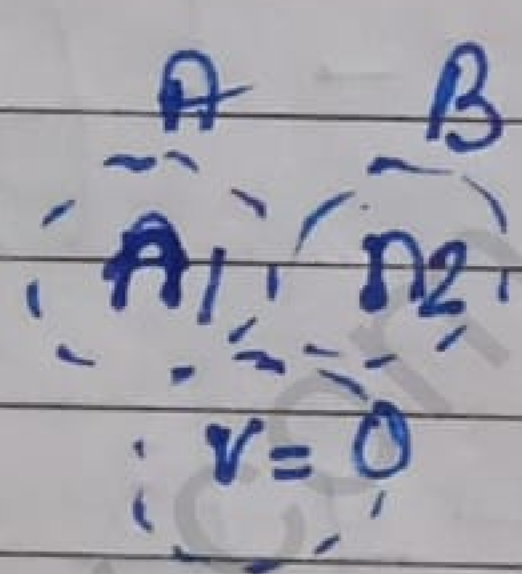
B. Collision



$v_1 \neq v_2$, $K_1 = \frac{1}{2} m_1 v_1^2$ $\qquad K_2 = \frac{1}{2} m_2 u^2$

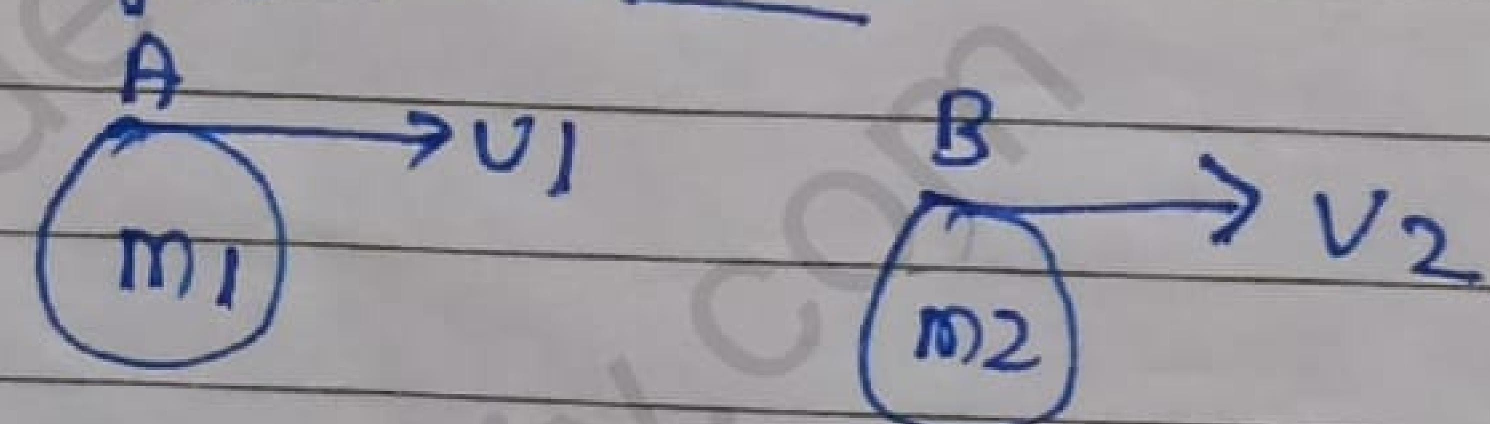
$m_1 \neq m_2$ $\quad P_1 = m_1 v_1$ $\qquad P_2 = m_2 v_2$

Case 2



Case 3

After Collision



$K_1 = \frac{1}{2} m_1 v_1^2$ $\qquad K_2 = \frac{1}{2} m_2 v_2^2$

$P_1 = m_1 v_1$ $\qquad P_2 = m_2 v_2$

From Linear momentum Conservation

$$P_{B.C.} = P_{A.C.}$$

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$
$$m_1u_1 - m_1v_1 = m_2v_2 - m_2u_2$$
$$\therefore \quad m_2(v_2 - u_2) = m_1(v_1 - u_1) \quad - (1)$$

Similarly K.E. Conservation Law

$$K.E_{BC} = KE_{AC}$$

$$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$$

$$m_1u_1^2 + m_2u_2^2 = m_1v_1^2 + m_2v^2$$
$$m_2v_2^2 - m_2u_2^2 = m_1u_1^2 - m_1v_1^2$$
$$m_2(v_1^2 - u_2^2) = m_1(v_1^2 - u_1^2) \quad - (2)$$

from above discussion we are getting two equation so eq(2)/eq(1)

$$\frac{\cancel{m_2}(v_1^2 - u_2^2)}{\cancel{m_2}(v_2 - u_2)} = \frac{\cancel{m_1}(u_1^2 - v_1^2)}{\cancel{m_1}(u_1 - v_1)}$$

$$\frac{v_2^2 - u_2^2}{v_2 - u_2} = \frac{u_1^2 - v_1^2}{u_1 - v_1}$$

$$\frac{(v_2 + u_2)\cancel{(v_2 - u_2)}}{\cancel{(v_2 - u_2)}} = \frac{(u_1 + v_1)\cancel{(u_1 - v_1)}}{\cancel{(u_1 - v_1)}}$$

Evergreen
Page No.
Date: / /20

$$v_2 + u_2 = u_1 + v_1 \quad - (3)$$

$$v_1 = v_2 + u_2 - v_1 \quad - (4)$$

$$v_2 = u_1 + v_1 - u_2 \quad - (5)$$

Now putting the value of v_1 from eq (4) into eq(1) we are getting the value of v_2

$$m_2 (v_2 - u_2) = m_1 (u_1 - v_1)$$

$$m_2 (v_2 - u_2) = m_1 (u_1 - v_2 + u_2 - v_1)$$

$$m_2 v_2 - m_2 u_2 = 2m_1 u_1 - m_1 v_2 + m_1 u_2 \cancel{- m_1 v_1}$$

$$m_2 v_2 - m_2 u_2 = 2m_1 u_1 - m_1 v_2 - m_1 u_2$$

And again putting the value of v_2 eq (5) into eq (1) getting the value of v_1

$$m_2 (v_2 - u_2) = m_1 (u_1 - v_1)$$

$$m_2 (u_1 + v_1 - u_2 - u_2) = m_1 (u_1 - v_1)$$

$$m_2 u_1 + m_2 v_1 - 2m_2 u_2 = m_1 u_1 - m_1 v_1$$

$$m_2 v_1 + m_1 v_1 = m_1 u_1 - m_2 u_1 + 2m_2 u_2$$

$$v_1 (m_1 + m_2) = (m_1 - m_2) u_1 + 2m_2 u_2$$

$$v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) u_1 + \frac{2m_2 u_2}{m_1 + m_2}$$

$$m_2 v_2 + m_1 v_2 = 2m_1 u_1 - m_1 v_2 - m_1 u_2$$

$$v_2 (m_2 + m_1) = 2m_1 u_1 + u_2 (m_1 - m_2)$$

$$v_2 = \frac{2m_1 u_1 (m_1 - m_2)}{(m_1 + m_2)}$$

$$v_1 = \frac{2m_2 u_2 (m_2 - m_1)}{m_1 + m_2}$$

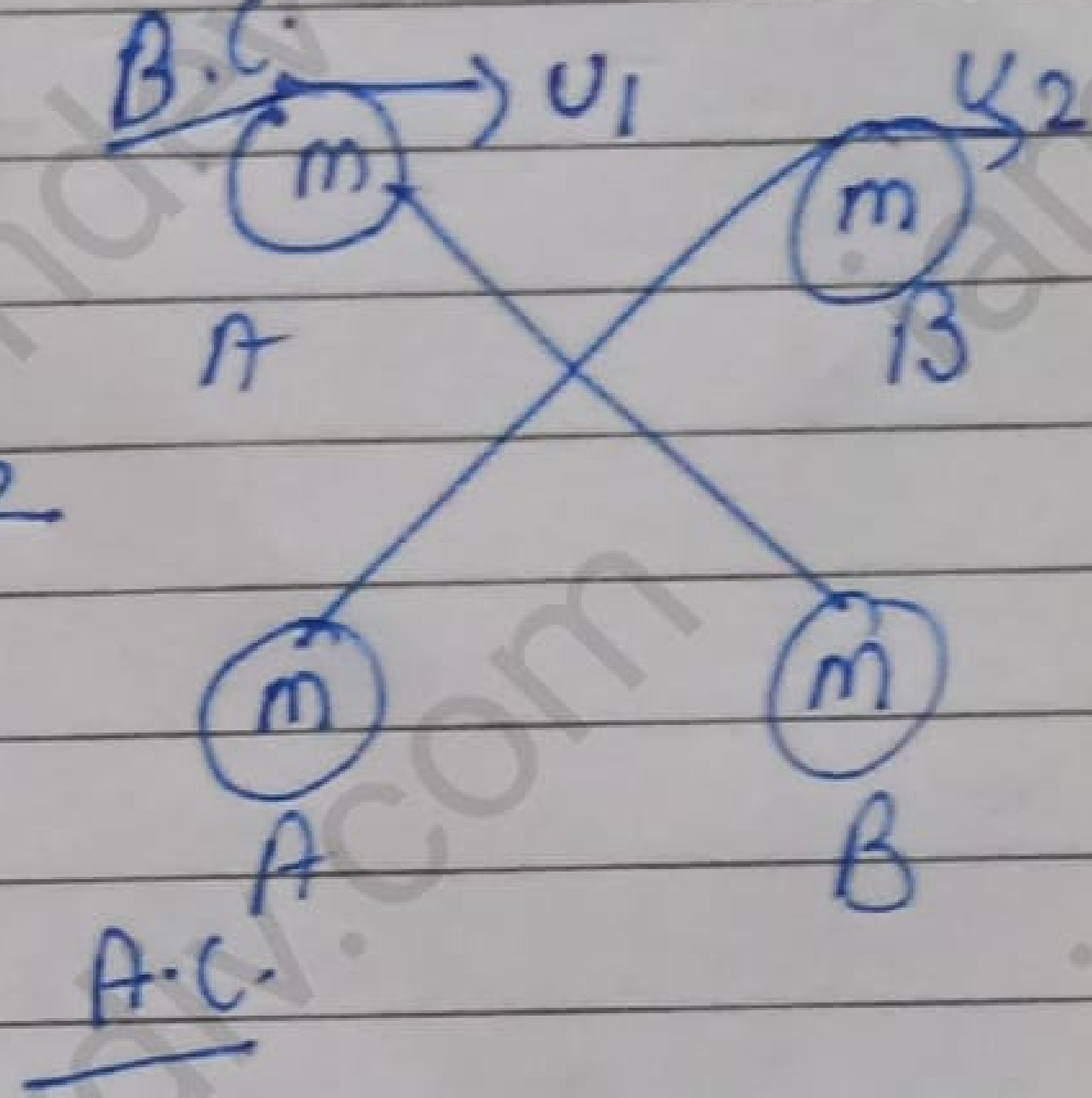
Case I - $v_1 = \frac{(m_1 - m_2) u_1}{m_1 + m_2} + \frac{2 m_2 u_2}{m_1 + m_2}$ — (1)

$v_2 = \frac{(m_2 - m_1) u_2}{m_1 + m_2} + \frac{2 m_1 u_1}{m_1 + m_2}$

If (1) $m_1 = m_2 = m$

$v_1 = 0 + \frac{2 m u_2}{\cancel{2m}} = u_2$

$v_2 = 0 + \frac{2 m u_1}{\cancel{2m}} = u_1$



Case 2 - $m_1 \neq m_2$

u_1 ✓ $u_2 = 0$

$v_1 = \frac{(m_1 - m_2) u_1}{m_1 + m_2} + 0$

$v_2 = 0 + \frac{2 m_1 u_1}{\cancel{2m}}$

Case 3 - v_1 ✓ $u_2 = 0$ $m_1 = m_2$

$v_1 = 0$ $v_2 = u_1$

Page No.
Date : / /20

Ans – $t = \sqrt{x} + 3$

$\downarrow$ Time $\downarrow$ displacement

$x \rightarrow$ displacement $v = 0$

$$t = \sqrt{x} + 3$$
$$t - 3 = \sqrt{x}$$
$$\sqrt{x} = t - 3$$
$$x = (t-3)^2$$
$$= t^2 + 9 - 6t$$
$$x = t^2 - 6t + 9 \quad \text{— (1)}$$

$$v = \frac{dx}{dt}$$

$$x = t^2 - 6t + 9$$
$$\frac{dx}{dt} = \frac{d}{dt}(t^2 - 6t + 9)$$

$$v = \frac{d}{dt}t^2 - 6\frac{d}{dt}t + \frac{d}{dt}9$$
$$= 2t - 6 + 0$$
$$\boxed{v = 2t - 6}$$

Good

if $v = 0$
$$2t - 6 = 0$$
$$\boxed{t = 3}$$

putting (1)
$$x = (3)^2 - 6 \times 3 + 9$$
$$x = 0$$

Ans. Work done by the force in 6 sec.

$$V = 2t - 6$$

$$a = \frac{dv}{dt} = 2$$

$$\boxed{x = t^2 - 6t + 9}$$

$$W = F \Delta x$$
$$= ma \Delta x$$

$t = 0$ to 6

$\cancel{t=0}$ $\quad x_1 = 9$

$t = 6$ $\quad x_2 = (6)^2 - 6 \times 6 + 9$

$$= 36 - \cancel{36}\ 36 + 9$$

$$x_2 = 9$$

$$\Delta x = x_2 - x_1 = 9 - 9 = 0$$

$$W = ma \Delta x$$
$$= ma \times 0$$
$$= 0$$

Seen

07/09/23